

Section—C

(Long Answer Type Questions)

Note : Attempt any two questions. Each question carries 12 marks.

6. Obtain Fourier's series of $f(x) = x \sin x$ in the interval $(-\pi, \pi)$. Hence deduce that :

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots$$

7. (a) Solve :

$$x \frac{dy}{dx} + y = y^2 \log x$$

- (b) Solve :

$$(x^2 - ay) dx - (ax - y^2) dy = 0$$

8. Solve :

$$\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} - y = xe^x + e^x$$

9. Solve :

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = 8x^2 e^{2x} \sin 2x$$

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BCA-405(N)

B. C. A. (Fourth Semester)
EXAMINATION, May, 2019

(New Course)

Paper Fifth

MATHEMATICS-III

Time : Three Hours] [Maximum Marks : 75

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : All questions are compulsory. Each question carries 3 marks.

1. (A) Find real numbers x and y , if :

$$x + iy = \frac{2 - 3i}{7 + 4i}$$

- (B) Define monotonic sequence and explain it with example.

- (C) If $\vec{r} = (\cos nt)\hat{i} + (\sin nt)\hat{j}$, then show that :

$$\vec{r} \cdot \frac{d\vec{r}}{dt} = 0$$

- (D) If $\vec{F} = x^2z\hat{i} - 2y^3z^2\hat{j} + xy^2z\hat{k}$, find $\text{div } \vec{F}$ at $(1, -1, 1)$.

- (E) Simplify $(6+i) + (7-i)^2 - (3-4i)^2$ and write in the form $a + ib$.

- (F) Find the half-range cosine series for $f(x) = 1$ in $0 < x < \pi$.

- (G) Solve the differential equation :

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

- (H) Solve :

$$(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$$

- (I) Solve :

$$\frac{d^2y}{dx^2} - 7 \frac{dy}{dx} + 12y = 0$$

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Section—B

(Long Answer Type Questions)

Note : Attempt any two questions. Each question carries 12 marks.

2. (a) Find all value of $(1+i)^{\frac{1}{3}}$.

- (b) Solve the equation :

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$$

3. (a) Prove that :

$$\lim \left[\left(\frac{2}{1}\right) \cdot \left(\frac{3}{2}\right)^2 \cdot \left(\frac{4}{3}\right)^3 \cdots \left(\frac{n+1}{n}\right)^n \right]^{\frac{1}{n}} = e$$

- (b) Discuss the convergence of the series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$$

4. Find the directional derivative of :

$$f(x, y, z) = x^2yz + 4xz^2$$

at the point $(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.

5. Test for convergence the series :

$$\sum_{n=1}^{\infty} \frac{3.6.9 \cdots 3n}{7.10.13 \cdots (3n+4)} x^n, (x > 0)$$

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